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β - DECAY MATRIX ELEMENTS IN STRONGLY DEFORMED NUCLEI

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SUMMARY

In this paper* we present the calculation of the β-decay matrix elements in strongly deformed nuclei. We have used the wave functions of A. Bohr and B. R. Mottelson for describing the collective properties of nuclei and S. G. Nilsson's wave function for the individual nucleons. The vibrational wave functions are left unspecified. The matrix elements are given for odd A nuclei. For the relativistic operators we have used the form obtained by a Foldy — Wouthuysen transformation. Some of the matrix elements calculated here have been obtained also independently by S. G. Nilsson.

I. *Introduction*

The theory of β-decay as formulated originally by E. Fermi and developed later by E. Konopinski-G. Uhlenbeck, E. Creuling [1] and many others is relativistic. Relativistic invariant expressions are built from the spinor fields of nucleons (protons, neutrons) and leptons (electron, neutrino). To apply it to complex nuclei the nucleons are treated in configuration space and the effects of pair formation of the nucleons are neglected.

The β-spectra, half lives and electron neutrino angular correlations are calculated by the perturbation method. The nuclear matrix elements are treated as parameters.

To determine the strenght of the coupling constants one can — in the simplest cases — calculate the matrix elements and determine from the measured data the coupling constants. Such investigations have been performed for decay of mirror [2] nuclei, namely, nuclei for which the initial and the final state differ only by the Coulomb effect. The shell model seems to work well in such cases because the effect of the core structure seems to be negligible. Calculations of beta matrix elements with shell model wave functions have been performed by many authors [3]. They have used the nonrelativistic

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approximation of relativistic matrix elements containing α and γ_5 and one and many particle shell model wave functions [4]. Contrary to the case of mirror nuclei the corrections of many particle configurations are in many cases very important. That means that the core structure in such cases has to be taken into account. A further correction may arise from the configuration mixing [5]. Shell model neglects also the effect of residual interactions [6], namely, those which are not included in the average potential which may also play an important role.

An improvement of the shell model is obtained if one takes into account the dynamical effects of the core and couples them to the particle motion. Such a model has been proposed and elaborated by A. Bohr and R. R. Mottelson [7]. In the region between closed shells as a consequence of strong coupling between particles and the core the nuclei have large equilibrium deformations. The motion of a nucleon in such a deformed field has been studied by S. G. Nilsson [8] who has computed the wave function for such a model. Using these wave functions — since they include only the average effects of the nuclear dynamics — one might hope to obtain some general trends but not detailed description of the fluctuation of ft values. Such qualitative effect is obtained indeed in the study of the asymptotic behaviour of these functions for large deformation [9]. That may give us some hope and encouragement for more quantitative analysis.

The purpose of this paper is to compute the β-matrix elements for strongly deformed nuclei and give the possibility for a quantitative analysis.

II. *Operators*

Let us start with the five interactions (scalar, vector, tensor, pseudovector and pseudoscalar) used in the β-decay. We write them in the configuration space of the nucleons, and for the lepton part which is taken at the position of the transforming nucleon we use spherical tensor representation [10]:

$$H_1 = \beta L_1 \tag{1}$$

$$H_2 = L_2 - \vec{\alpha}\,\mathbf{L}_2, \tag{2}$$

$$H_3 = \beta\,\vec{\sigma}\,\mathbf{L}_3 + \beta\,\vec{\alpha}\,\mathbf{L}_3', \tag{3}$$

$$H_4 = \vec{\sigma}\,\mathbf{L}_4 - \gamma_5 L_4, \tag{4}$$

$$H_5 = \beta\,\gamma_5 L_5. \tag{5}$$

The linear combination of the five covariants is

$$H_\beta = g_\beta \sum_{i=1}^{5} g_i\,[H_i + H_i^*]. \tag{1a}$$

β, $\vec{\sigma}$, $\vec{\alpha}$ and γ_5 are the usual Dirac matrices of the nucleons and L_1, L_2 etc. are the obvious abbreviations for lepton covariants [10]. g_i are the relative coupling constants of the five covariants. g_β measures the strength of the beta coupling.

The nucleon operators β and $\vec{\sigma}$ do not mix the large and the small components of the nucleon wave functions and are of the order of magnitude of unity. $\vec{\alpha}$ and γ_5 are called the relativistic operators because they mix the large and the small components and are — roughly speaking — of the order of magnitude of v/c where v is the average velocity of the nucleon in the nucleus and c the velocity of light.

The main terms of the nonrelativistic form of the relativistic operators as obtained by a Foldy-Wouthuysen transformation [11] may be written [10]:

$$\vec{\alpha} = \frac{1}{M}\left(-\mathbf{P} + i\,\mathbf{V}_2 + i\,\vec{\sigma}\times\mathbf{V}_3' - \vec{\sigma}\cdot V_4\right) - \frac{1}{2M}\left[i\,\vec{\sigma}\times\mathbf{P} + \mathbf{P},\right], \tag{6}$$

$$\beta\,\vec{\alpha} = -\frac{1}{M}\left(i\,\vec{\sigma}\times\mathbf{P} + \vec{\sigma}\times\mathbf{V}_2 - \mathbf{V}_3' - V_5\,\vec{\sigma}\right) - \frac{1}{2M}\left[i\,\vec{\sigma}\times\mathbf{P} + \mathbf{P},\right], \tag{7}$$

$$\gamma_5 = -\frac{1}{M}\left(\vec{\sigma}\cdot\mathbf{P} - i\,\vec{\sigma}\cdot\mathbf{V}_2 + i\,V_4\right) - \frac{1}{2M}\left[\vec{\sigma}\,\mathbf{P},\right], \tag{8}$$

$$\beta\,\gamma_5 = \frac{1}{M}\left(\vec{\sigma}\cdot\mathbf{V}_3' + V_5\right) - \frac{1}{2M}\left[\vec{\sigma}\,\mathbf{P},\right]. \tag{9}$$

$\mathbf{V}_2$, $\mathbf{V}_3$ etc. represent the average potentials in which a nucleon is assumed to move. The dominant terms seem to be the first terms in the paranthesis except, perhaps, in the case of pseudoscalar coupling. Using these terms and the spherical tensor representation of lepton covariants we obtain the following interaction operators:

$$H_1 = \beta\,\varphi_1(\Lambda, \varkappa, \varkappa_\nu, r, p, q)\, r^\Lambda\, Y_\Lambda^{-M}, \tag{10}$$

$$H_2 = \varphi_2\, r^\Lambda\, Y_\Lambda^{-M} + \varphi_2'\,\frac{1}{M}\, r^\Lambda\, \mathbf{Y}_J^{-M}\cdot\mathbf{P}, \tag{11}$$

$$H_3 = \beta\,\varphi_3\, r^\Lambda\, \vec{\sigma}\,\mathbf{Y}_{J\Lambda}^{-M} - \varphi_3'\,\frac{1}{M}\, r^\Lambda\, \mathbf{Y}_{J\Lambda}^{-M}\,\vec{\sigma}\times\mathbf{P}, \tag{12}$$

$$H_4 = \varphi_4\, r^\Lambda\, \vec{\sigma}\cdot\mathbf{Y}_{J\Lambda}^{-M} + \varphi_4'\,\frac{1}{M}\, r^\Lambda\, Y_\Lambda^{-M}\,\vec{\sigma}\cdot\mathbf{P}, \tag{13}$$

$$H_5 = \varphi_5\,\frac{1}{M}\, r^\Lambda\, Y_\Lambda^{-M}\left(\vec{\sigma}\cdot\mathbf{V}_3' + V_5\right) - \frac{1}{2M}\left[\vec{\sigma}\cdot\mathbf{P}, \varphi_5\, r^\Lambda\, Y_\Lambda^{-M}\right]. \tag{14}$$

$\varphi_1(\Lambda, \varkappa, \varkappa_\nu, r, w)$ etc. are the functions dependent on the angular momenta position and momenta of the leptons and their couplings. For the definition of this functions see ref. [10] formula (64) and

(65). From the formula (10)—(14) it is seen that one has the following type of operators in the β-decay [12]:

$$r^{\Lambda}\, \mathbf{Y}_{\Lambda}^{-M}\,, \tag{15}$$

$$r^{\Lambda}\, \vec{\sigma} \cdot \mathbf{Y}_{J\Lambda}^{-M} = r^{\Lambda}\, T_{J\Lambda}^{-M}(\mathbf{r}, \vec{\sigma}), \tag{16}$$

$$r^{\Lambda}\, \mathbf{Y}_{J\Lambda}^{-M} \cdot \mathbf{P} = -\,i\, r^{\Lambda}\, T_{J\Lambda}^{-M}(\mathbf{r}, \nabla), \tag{17}$$

$$r^{\Lambda}\mathbf{Y}_{J\Lambda}^{-M} \cdot \vec{\sigma} \times \mathbf{P} = -\,i\, r^{\Lambda}\, T_{J\Lambda}^{-M}(\mathbf{r}, \vec{\sigma} \times \nabla), \tag{18}$$

$$r^{\Lambda}\, \mathbf{Y}_{J\Lambda}^{-M} \cdot \vec{\sigma} \cdot \mathbf{P} = -\,i\, r^{\Lambda}\, Y_{J\Lambda}^{-M}\; (\vec{\sigma} \cdot \nabla), \tag{19}$$

The angular part of the operators (15) and (19) are the ordinary spherical harmonics while those of (16), (17) and (18) are the tensor operators defined by

$$T_{J\Lambda}^{-M}(\mathbf{r}, \mathbf{a}) = \sum_{\nu\,\mu'} (1 - \nu\, \Lambda - M' \,|\, 1 \Lambda J - M)\, a_1^{-\nu}\, Y_{\Lambda}^{-M'}. \tag{20}$$

a_1^{ν} are the spherical components of the vector $\mathbf{a}$, and

$$(1 - \nu\, \Lambda - M' \,|\, 1\, \Lambda\, J - M)$$

are the vector addition coefficients as given in TAS [13].

It is worth noting the relation

$$T_{11}{}^{\mu}(\mathbf{a}, \mathbf{b}) = \sum_{\nu\nu'} (1\,\nu\, 1\, \nu' \,|\, 11\, 1\, \mu)\, a_1^{\nu}\, b_1^{\nu'} = \frac{i}{\sqrt{2}}\, (\mathbf{a} \times \mathbf{b})_1^{\mu}. \tag{21}$$

With the help of (21) it is possible to write the tensor operator (18) in the form

$$T_{J\Lambda}^{-\mu}(\mathbf{r}, \vec{\sigma} \times \nabla) = \sum_{\mu\, m} (1 - \mu\, \Lambda - m \,|\, 1\, \Lambda\, J - M)\, Y_{\Lambda}^{-m}\, (\vec{\sigma} \times \nabla)_1^{-\mu} =$$

$$= i\sqrt{2} \sum_{\substack{\mu\, m \\ \nu\nu'}} (1 - \mu\, \Lambda - m \,|\, 1\, \Lambda\, J - M)\, (1 - \nu\, 1 - \nu' \,|\, 111 - \mu)\, \mathbf{Y}_{\Lambda}^{-m}\, \sigma_1^{-\nu}\, \nabla_1^{-\nu'} \tag{22}$$

which facilitates the calculation of the matrix elements.

III. *The Model and the Wave Functions*

Because we concentrate our attention to the region of strongly deformed nuclei we shall be interested mainly in the collective model [7].

That model takes advantage of the collective properties of the nuclei and their coupling to the particle motion. The Hamiltonian for that case may be written in the form

$$H = H_{\mathrm{coll}}\,(\alpha, \dot{\alpha}) + H_p + H_{\mathrm{int}}. \tag{23}$$

The first term represents the collective part of the Hamiltonian dependent on collective coordinates and velocities. The second part is the particle Hamiltonian. The third term represents the interaction of the collective and particle motion. It is possible to obtain solutions of (23) in the case of weak and strong coupling.

The first case is met in the region of closed shells where only a few particles are outside closed shells.

Nuclear dynamics in this case is represented by the vibration of the core around equilibrium shape and motion of the particle. The interaction of the nuclear vibrations and particle motion is extremely weak so that it can be treated as a small perturbation.

In the case of strong coupling the interaction between core vibrations and particle motion is rather strong, and we have to use an approximation similar to that of Born and Oppenheimer [14] in molecular physics. It is actually applicable only if the collective frequencies are much smaller than the particle frequencies so that the particle is able to follow adiabatically the change of the nuclear field [7].

The properly symmetrised wave functions in this case may be written in the form

$$|\Omega\,;\,u_{\mathrm{vibr}}\,;\,I\,M\,K> = \sqrt{\frac{2\,I+1}{16\,\pi^2}}\,\varphi_{\mathrm{vibr}}\Big\{\chi_{\Omega}\,D^I{}_{MK}(\Theta_i) + (-)^{I-\sum_p j_p}\,\chi_{-\Omega}\,D^I{}_{M-K}(\Theta_i)\Big\} \tag{24}$$

$D^I{}_{M-K}(\Theta_i)$ and φ_{vibr} represent the collective rotations and vibrations, respectively. Θ_i are the Eulerian angles specifying the position of the body fixed system. I is the total angular momentum of the nucleus while M and K are the projections of I on the space and body fixed system, respectively. χ_{Ω} and $\chi_{-\Omega}$ are the particle wave functions.

Particle wave functions have been calculated by S. G. Nilsson [8] for the Hamiltonian:

$$H = -\frac{\hbar^2}{2M}\,\Delta + \frac{M}{2}\,(\omega_x^2\,x^2 + \omega_y^2\,y^2 + \omega_z^2\,z^2) + C\,\mathbf{l}\,\mathbf{s} + D\,\mathbf{l}^2. \tag{25}$$

The first part represents the Hamiltonian of an anisotropic harmonic oscillator with frequencies ω_x etc. C and D are the constants measuring the magnitude of the spin-orbit coupling and the interpolation term, respectively. M is the nucleon mass.

The wave functions are

$$\begin{gathered}\chi_{\Omega} = \sum_{l\Sigma} a_{l\,\Omega-\Sigma}\,R_{N\,l}\,Y_l{}^{\Omega-\Sigma}\,\chi_{1/2}{}^{\Sigma}\\ (-)^{I-\sum_p j_p}\,\chi_{-\Omega} = (-)^{I-\frac{1}{2}+l}\sum_{l\Sigma} a_{l\Omega-\Sigma}\,R_{Nl}\,Y_l{}^{-\Omega+\Sigma}\,\chi_{1/2}{}^{-\Sigma} = \chi_{\bar{\Omega}}\,.\end{gathered} \tag{25}$$

$a_{l\,\Omega-\Sigma}$ are the normalized amplitudes given in the tables by S. G. Nilsson [8] while

$$R_{N\,l}(r) = N_{N\,l}\,\varrho^{\frac{1}{2}l}\,e^{-\frac{1}{2}\varrho}\,L_n{}^a(\varrho) \tag{26}$$

is the normalized radial wave function. $L_n^a(\varrho)$ is the associated Laguerre polynomial. n and a are connected with the quantum numbers N and l by means of the relations

$$a = l + \frac{1}{2},$$
$$n = \frac{N-l}{2}. \tag{27}$$

r means $a\sqrt{\varrho}$ and N_{Nl} is the normalization factor

$$N_{Nl} = \sqrt{\frac{\Gamma(n+1)}{\Gamma(n+a+1)^3}}\sqrt{\frac{2}{a^3}}. \tag{27'}$$

$$a = \sqrt{\frac{\hbar}{\omega_0 M}} = 1{\cdot}2 \times 10^{-13} A^{1/3} \sqrt{\frac{6}{5(2N+3)}}\ \text{cm} \tag{28}$$

or in $\frac{\hbar}{mc}$ unit

$$a = \frac{\sqrt{\frac{6}{5}} \times 1{\cdot}2 \times 2{\cdot}6}{\sqrt{2N+3}} \times 10^{-3} A^{1/3} \tag{28'}$$

is determined from the requirement that the expectation value of r^2 should be equal to the square of the average radius. ω_0 is given by means of the deformation δ

$$\omega_x^2 = \omega_y^2 = \omega_0^2\left(1 + \frac{2}{3}\delta\right), \tag{29}$$

$$\omega_z^2 = \omega_0^2\left(1 - \frac{4}{3}\delta\right).$$

δ is connected with the deformation parameter used by A. Bohr and B. R. Mottelson by

$$\delta = \frac{3}{2}\sqrt{\frac{5}{4\pi}}\,\beta \approx 0.95\,\beta \tag{30}$$

$Y_l^{\Omega-\Sigma}$ and $\chi_{1/2}^{\Sigma}$ are the spherical harmonics and the spin wave functions, respectively.

IV. *Matrix Elements*

Now we may start to calculate the matrix elements for the β-decay. Let us start with the operator $T^{\mu}_{JA}(\mathbf{r}, \vec{\sigma})$. First we have to transform the operator to the body fixed frame of reference

$$T^{\mu}_{JA}(\mathbf{r}, \vec{\sigma}) = \sum_{\mu'} D^{J}_{\mu\mu'}\, T^{\mu'}_{JA}(\mathbf{r}, \vec{\sigma}) \tag{31}$$

and we use the formula

$$\int D^{I'^*}_{MK}\, D^{J}_{\mu\mu'}\, D^{I'}_{M'K'}\, d\Omega = (-)^{-(K+M)}\, 8\pi^2 \begin{pmatrix} I & J & I' \\ -M & \mu & M' \end{pmatrix} \begin{pmatrix} I & J & I' \\ -K & \mu' & K' \end{pmatrix} \tag{32}$$

to obtain

$$\langle I \,|\, T_{JA} \,|\, I' \rangle_\beta = (-)^{-(I+K)} \sqrt{2I+1} \sum_{\mu'} \left\{ \begin{pmatrix} I & J & I' \\ -K & \mu' & K' \end{pmatrix} \cdot \right. \tag{33}$$

$$\left. \cdot \langle \Omega \,|\, T^{\mu'}_{JA} \,|\, \Omega' \rangle + \begin{pmatrix} I & J & I' \\ -K & \mu' & -K' \end{pmatrix} \langle \Omega \,|\, T^{\mu'}_{JA} \,|\, \overline{\Omega'} \rangle \right\} |F|^2.$$

The reduced matrix element is that used in the beta decay. $\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$ is the three $-j$ symbol [15]. $|F|^2$ is the vibrational matrix element.

The problem is reduced now to the calculation of matrix elements $\langle \Omega \,|\, T_J{}^{\mu} \,|\, \Omega' \rangle$ for different operators since formula (33) is independent of any special assumptions. Let us start with $T^{\mu'}_{JA}\,(\mathbf{r}, \vec{\sigma})$. A straightforward calculation gives

$$\langle \Omega \,|\, r^{A}\, T^{\mu'}_{JA}\,(\mathbf{r}, \vec{\sigma}) \,|\, \Omega' \rangle = (-)^{\Omega + s + l + A + \mu'} \sqrt{2J+1}\, \langle s \,\|\, \sigma_1 \,\|\, s' \rangle$$

$$\sum_{\substack{l l' \\ \Sigma \Sigma' m \varepsilon}} a_{l\,\Omega-\Sigma}\, a_{l'\,\Omega'-\Sigma'} \langle N\,l \,|\, r^{A} \,|\, N'\,l' \rangle \langle l \,\|\, Y_A \,\|\, l' \rangle \begin{pmatrix} 1 & A & J \\ \varepsilon & m & -\mu' \end{pmatrix} \cdot$$

$$\cdot \begin{pmatrix} l & A & l' \\ -A+\Sigma & m & \Omega'-\Sigma' \end{pmatrix} \cdot \begin{pmatrix} s & 1 & s' \\ -\Sigma & \varepsilon & \Sigma' \end{pmatrix}. \tag{34}$$

As special cases of the expression (34) we obtain

$$\langle \Omega \,|\, r^{A}\, Y^{\mu'}_{A} \,|\, \Omega' \rangle = (-)^{l} \sum_{l l' \Sigma} (-)^{-\Omega+\Sigma}\, a_{l\,\Omega-\Sigma}\, a_{l'\,\Omega'-\Sigma'}$$

$$\langle N\,l \,|\, r^{A} \,|\, N'\,l' \rangle \langle l \,\|\, Y_A \,\|\, l' \rangle \begin{pmatrix} l & A & l' \\ -\Omega+\Sigma' & \mu' & \Omega'-\Sigma' \end{pmatrix} \tag{34'}$$

$$\langle \Omega \,|\, \sigma_1{}^{\mu'} \,|\, \Omega' \rangle = (-)^{1/2} \sum_{\Sigma} (-)^{\Sigma}\, a_{l\,\Omega-\Sigma}\, a_{l\,\Omega'-\Sigma'} \langle s \,\|\, \sigma_1 \,\|\, s' \rangle \begin{pmatrix} s & 1 & s' \\ -\Sigma & \mu' & \Sigma' \end{pmatrix} \tag{34''}$$

the expressions for ordinary spherical harmonics and σ matrix.

The matrix elements for derivative couplings may be evaluated in the similar way

$$\langle \Omega \,|\, r^{A}\, T^{\mu'}_{JA}\,(\mathbf{r}, \vec{\nabla}) \,|\, \Omega' \rangle = (-)^{\mu'+1} \sum_{\substack{m \varepsilon'' \\ l l' \Sigma}} (-)^{m} \tag{35}$$

$$\sqrt{2J+1}\; a_{l\,\Omega-\Sigma}\; a_{l'\,\Omega'-\Sigma} \begin{pmatrix} 1 & A & J \\ \varepsilon'' & m & -\mu' \end{pmatrix} b\,(\Sigma = \Sigma'),$$

$$\langle \Omega \mid r^{\Lambda}\, Y_{\Lambda}^{m}\, \vec{\sigma}\, \vec{\nabla} \mid \Omega' \rangle = (-)^{\Lambda + s - \Omega} \sum_{\substack{l\, l' \\ \varepsilon'' \Sigma \Sigma'}} (-)^{\Omega' - \Sigma'}\, a_{l\, \Omega - \Sigma}$$

$$a_{l'\, \Omega' - \Sigma'} \langle s \| \sigma_1 \| s' \rangle \begin{pmatrix} s & 1 & s' \\ -\Sigma & -\varepsilon'' & \Sigma' \end{pmatrix} b, \tag{36}$$

$$\langle \Omega \mid r^{\Lambda}\, T_{J\Lambda}^{\mu'}\, (\mathbf{r}, \vec{\sigma} \times \vec{\nabla}) \mid \Omega' \rangle = i \sqrt{6\, (2\, J + 1)} \tag{37}$$

$$\sum_{\substack{l\, l'\, \Sigma\, \Sigma' \\ \varepsilon\, \varepsilon'\, \varepsilon''\, m}} (-)^{s - \Sigma} \begin{pmatrix} 1 & \Lambda & J \\ \varepsilon & m & -\mu' \end{pmatrix} \begin{pmatrix} 1 & 1 & 1 \\ \varepsilon' & \varepsilon'' & \varepsilon \end{pmatrix} \begin{pmatrix} s & 1 & s' \\ -\Sigma & \varepsilon' & \Sigma' \end{pmatrix}$$

$$a_{l\, \Omega - \Sigma}\, a_{l'\, \Omega' - \Sigma'} \langle s \| \sigma_1 \| s' \rangle\, b.$$

The abbreviations used are [15], [16]

$$\langle l \| Y_{\Lambda} \| l' \rangle = (-)^{l} \sqrt{\frac{(2\, l + 1)\, (2\, l' + 1)\, (2\, \Lambda + 1)}{4\, \pi}} \begin{pmatrix} l & \Lambda & l' \\ 0 & 0 & 0 \end{pmatrix} \tag{38}$$

$$\langle N\, l \mid r^{\Lambda} \mid N'\, l' \rangle = a^{\Lambda}\, \Gamma\, (p + 1) \sqrt{\frac{\Gamma\, (n + 1)\, \Gamma\, (n' + 1)}{\Gamma\, (n + \alpha + 1)\, \Gamma\, (n' + \alpha' + 1)}}\, \cdot$$

$$\cdot \sum_{\tau} \binom{p - \alpha}{n - \tau} \binom{p - \alpha'}{n' - \tau} \binom{p + \tau}{\tau}. \tag{39}$$

The sumation has to be extended over all nonvanishing binomial coefficients.

Γ is the ordinary gamma function

$$p = \frac{1}{2}\, (l + l' + \Lambda + 1), \tag{40}$$

$$\langle N\, l \mid r^{\Lambda}\, D \pm \mid N'\, l' \rangle = \Big\{ - \sqrt{\frac{n + 1}{n' + \alpha' + 2}} \langle N\, l \mid r^{\Lambda} \mid N'\, (n' + 1)\, l' \rangle$$

$$- (1 + \alpha') \langle N\, l \mid r^{\Lambda} \mid N'\, l' \rangle + \tag{41}$$

$$+ (n' + \alpha' + 1) \sqrt{n'\, (n' + \alpha')} \langle N\, l \mid r^{\Lambda} \mid N'\, (n' - 1)\, l' \rangle \Big\} \frac{1}{a}$$

$$p = \frac{1}{2}\, (l' + l + \Lambda) \tag{42}$$

When using formula (41) $(n' + 1)$ and $(n' - 1)$ has to be supstituted in (39) with (42) for p

$$\langle s \| \sigma_1 \| s' \rangle = 2 \sqrt{s\, (s + 1)\, (2\, s + 1)}. \tag{43}$$

The expression for b is as follows

$$b = \sqrt{l' + 1}\begin{pmatrix} l' & 1 & l' + 1 \\ \Omega' - \Sigma' & \varepsilon'' & -\Omega' + \Sigma' - \varepsilon'' \end{pmatrix}\begin{pmatrix} l & \Lambda & l' + 1 \\ -\Omega + \Sigma & m & \Omega' - \Sigma' + \varepsilon'' \end{pmatrix}$$

$$\langle N\, l \,|\, r^{\Lambda} D_- \,|\, N'\, l' \rangle \, \langle l \,\|\, Y_{\Lambda} \,\|\, l' + 1 \rangle - \sqrt{l'} \tag{44}$$

$$\begin{pmatrix} l' & 1 & l' - 1 \\ \Omega' - \Sigma' & \varepsilon'' & -\Omega' + \Sigma' - \varepsilon'' \end{pmatrix}\begin{pmatrix} l & \Lambda & l' - 1 \\ -\Omega + \Sigma & m & \Omega' - \Sigma' + \varepsilon'' \end{pmatrix} \cdot$$

$$\cdot \, \langle N\, l \,|\, r^{\Lambda} D_+ \,|\, N'\, l' \rangle \, \langle l \,\|\, Y_{\Lambda} \,\|\, l' - 1 \rangle \,.$$

The radial operators D_+ and D_- are

$$\begin{aligned} D_+ &= \frac{d}{dr} + \frac{l' + 1}{r}, \\ D_- &= \frac{d}{dr} - \frac{l'}{r}. \end{aligned} \tag{45}$$

In deriving (44) we used also the expression

$$\int Y_{l_1}^{m_1 *} Y_{l_2}^{m_2} \nabla_1^{\varepsilon} Y_{l_3}^{m_3} \, d\,\Omega = (-)^{l_2 + m_2} \Big\{ \sqrt{l_3 + 1} \tag{46}$$

$$\langle l_1 \,\|\, Y_{l_2} \,\|\, l_3 + 1 \rangle \begin{pmatrix} l_3 & 1 & l_3 + 1 \\ m_3 & \varepsilon & -m_3 - \varepsilon \end{pmatrix}\begin{pmatrix} l_1 & l_2 & l_3 + 1 \\ -m_1 & m_2 & m_3 + \varepsilon \end{pmatrix} D_- -$$

$$- \sqrt{l_3}\, \langle l_1 \,\|\, Y_{l_2} \,\|\, l_3 - 1 \rangle \begin{pmatrix} l_3 & 1 & l_3 - 1 \\ m_3 & \varepsilon & -m_3 - \varepsilon \end{pmatrix}\begin{pmatrix} l_1 & l_2 & l_3 - 1 \\ -m_1 & m_2 & m_3 + \varepsilon \end{pmatrix} D_+ \Big\} .$$

The associated Laguerre polinomials are defined by means of the generating functions as [8]

$$\sum_{n=0}^{\infty} \frac{(-t)^n}{\Gamma(n + a + 1)} \, L_n^{a}(\varrho) = \frac{e^{-\frac{\varrho t}{1-t}}}{(1 - t)^{a+1}} \,. \tag{47}$$

It is important to mention that the second part of the equation (33) is important and gives corrections for the allowed and the first forbidden transitions in the case that the transitions are of the form $\frac{1}{2} \to \frac{1}{2}$ and $\frac{1}{2} \to \frac{3}{2}$. For higher forbidden transitions the correction is unimportant.

By means of the second equation of (25′) we obtain for the expression corresponding to (34)

$$\langle \Omega \,|\, T^{\mu'}_{J\Lambda} \,|\, \Omega' \rangle = (-)^{l' + \Omega + \mu'} \sqrt{2J+1}\, \langle s \,\|\, \sigma_1 \,\|\, s' \rangle \,\cdot$$
$$\sum_{\substack{l l' \\ \Sigma \Sigma' m \varepsilon}} a_{l\, \Omega - \Sigma}\, a_{l'\, \Omega' - \Sigma'}\, \langle N l \,|\, r^{1} \,|\, N' l' \rangle\, \langle l \,\|\, Y_{\Lambda} \,\|\, l' \rangle \,\cdot \tag{48}$$
$$\cdot \begin{pmatrix} 1 & \Lambda & J \\ \varepsilon & m & -\mu' \end{pmatrix} \begin{pmatrix} l & \Lambda & l' \\ -\Omega + \Sigma & m & -\Omega' + \Sigma' \end{pmatrix} \begin{pmatrix} s & 1 & s' \\ -\Sigma & \varepsilon & -\Sigma' \end{pmatrix}.$$

Comparing (48) with (34) it is obvious that (48) is obtained from (34) by multiplying it by a phase factor $(-)^{l' + l' - 1/2}$ and changing the signs of Ω' and Σ'. Doing the same in all other cases we get the corrections for other matrix elements.

V. *f t Values*

The transition probability as a function of the electron energy is given by

$$N(W) = \frac{1}{2\pi^3}\, W\, p\, q^2\, F\, C_\beta\,. \tag{49}$$

W and p are the energy and the momentum of the electron. q is the momentum of the neutrino. F is the well-known Fermi function which measures the influence of the Coulomb field on the spectrum. C_β is the so called correction factor which may be written in the form

$$C_\beta = \frac{4\pi^2}{p^2 q^2 F} \sum_{\mu \mu_\nu} \sum_{f} \int_i \left| \int \Psi_f^* H_\beta \Psi_i \, d\tau_1 \ldots d\tau_A \right|^2. \tag{50}$$

H_i is the interaction (1 a). The summation has to be performed over the angular quantum numbers of electron and neutrino. f and i refers to the final and initial states of the nucleus, respectively. We take the average over the initial and sum over final states of the nucleons. The average life time τ and the half life t are given by the energy or momentum integral of (49).

$$\frac{1}{\tau} = \frac{\ln 2}{t} = \int N(W)\, dW = \frac{1}{2\pi^3} \int (W_0 - W)^2\, p^2\, F\, C_\beta\, dp. \tag{51}$$

The function $p^2 F$ is tabulated for several values of p and z. There are also tables for the integral of $(W_0 - W)^2 p^2 F$ for several values of z and maximum energy W_0.

$$f\, t = t \int (W_0 - W)^2\, p^2\, F\, dp, \tag{52}$$

i. e. the product of the measured half life in seconds and the function f is called the $f\,t$ value. Because the half lifes of β-emitters are spread over a large range of values the quantity $f\,t$ corrected for the maximum energy is expected to be more suitable in analysis of the experimental data.

It is simple to work out the expression for $f\,t$ value if one uses the theoretical value for t

$$f\,t = \frac{l\,n\,2}{\int N(W)\,d\,W} \approx \frac{2\,\pi^3\,l\,n\,2}{\overline{C}_\beta}.$$

The last step is justified if the correction factor does not depend very much on energy. $\overline{C}_\beta$ is the abbreviation for the average value of the correction factor.

In this way we have completed the expressions necessary for calculating $f\,t$ values and comparing them with experiments.

An analysis of the experimental $f\,t$ values in the region of strongly deformed nuclei may be found in ref. [17]

At the end I would like to aknowledge the assistance of D. Zivkovic in the calculations.

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MATRICNI ELEMENTI β-RASPADA JAKO DEFORMIRANIH JEZGRI

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Sadrzaj

U ovoj radnji su izracunati matricni elementi za beta-prijelaze kod jako deformiranih jezgri. Nerelativisticki oblik relativistickih operatora dobiven je Foldy-Wouthuysenovom transformacijom, a valne funkcije su one, koje su dane kod A. Bohra-B. R. Mottelsona i S. G. Nilssona.

Poznato je da se u teoriji beta-radioaktivnog raspadanja matricni elementi, koji ovise o strukturi jezgre, tretiraju kao parametri. Usporedbom racunatih i mjerenih velicina dobivamo izvjesne obavijesti o velicini ovih parametara. Jasno, da bi to mogli naciniti, moramo poznavati zakone beta-radioaktivnog raspadanja. Kako njih međutim moramo isto tek odrediti, to postupak ide u stvari kombinirano. Za slucajeve, gdje se matricni elementi daju jednostavno odrediti, mogu se dobiti izvjesne obavijesti o konstantama vezanja. To se onda primjenjuje dalje da odredimo matricne elemente u drugim kompliciranijim slucajevima. To je dakle kao neka vrsta iteracije.

Kako su međutim kod nuklearnih modela matricni elementi oni, koji mnogo kriticnije ovise o detaljima nuklearne strukture, nego staticke velicine, energije i momenti, to ce racunanje matricnih elemenata i usporedba sa iskustvom biti daleko veca provjera ispravnosti modela. Kolektivni model A. Bohra i B. R. Mottelsona (23) i (24) imao je upravo zapanjujuce uspjehe u tumacenju nisko pobuđenih t. zv. rotacionih stanja, velikih kvadrupolnih momenata te ubrzanih kvadrupolnih i usporenih magnetskih dipolnih prijelaza. Racuni S. G. Nilssona i B. R. Mottelsona pokazali su nadalje, da se i cesticna stanja mogu klasificirati na temelju sheme dobivene rjesenjem gibanja jedne cestice u deformiranom potencijalu (25). Kvalitativno tumacenje usporenja elektricnih dipolnih prijelaza i nekih dozvoljenih i zabranjenih beta-prijelaza daje nade, da ce se dobiti i bolje slaganje za matricne elemente.

Transformirajuci operatore (31) u sistem cvrsto vezan sa jezgrom (cije su osi u glavnim osima tromosti) uz pomoc formula (32), mozemo dobiti izraze za matricne elemente (34), (34'), (34''), (35), (36) i (37).

Namjesto neposrednog uspoređivanja matricnih elemenata sa onima određenim iz mjerenja, puno je zgodnije usporedbu provesti na ft vrijednostima. ft vrijednosti su dane u zadnjem poglavlju.

Usporedbu sa iskustvom nacinit cemo u jednoj narednoj publikaciji.

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